Exploring the Solar System?

Let the Math Teachers Help!

by Karen Charles, J.D. Canales, Angela Smith, and Natalie Zimmerman

Scale measurement and ratio and proportion are topics that fall clearly in the middle-grades mathematics curriculum in our state, Texas. So does the solar system. In addition, the National Science Education Standards include the solar system in Earth and space science in the 5–8 grade band and promote the coordination of the science and mathematics curricula in Program Standard C (NRC 1996). In its chapter on Earth and space sciences, the new *Framework for K–12 Science Education* (NRC 2011) references Earth's place in the solar system and notes that these sciences "involve phenomena that range in scale from the unimaginably large to the invisibly small" (p. 121). But, as middle school mathematics teachers, we had never considered the connection until we participated in a weeklong academy offered by NASA this past summer in our school district. The academy encouraged middle and high school math and science teachers to consider how using models and simulations, or ModSim, could expand their repertoire of classroom strategies and engage students more fully in their own context-rich learning.

Through our work at the academy, we designed a cross-curricular lesson that incorporates the math skills we need to teach and the science concepts our colleagues teach their students, and uses ModSim to engage and challenge our students. We discovered that NASA was more than willing to help teachers like us upgrade our lessons with web-accessible materials and resources, many of which we will introduce the reader to in this lesson.

Scale, proportion, and ModSim

In our experience, we have found that students have trouble manipulating, much less comprehending, very large numbers and very small numbers. These concepts can be brought into students' realm of understanding through scale—scaling down very large objects and scaling up very small objects to develop models that can be seen and manipulated. Research by Schwarz and White (2005) found that activities requiring the creation and testing of models facilitated the learning of science concepts for the middle school students in their study.

A state map, for example, is a tool that students readily understand as a scaled-down model of a state's geography. Toys, such as trucks, trains, dolls, and action figures, are common examples of scaled-down models of real-world objects. So we assume that students understand the concept of scale, but we also know that the mathematics of scale, especially with large numbers, is a challenge in the middle grades. We believe that a concrete demonstration of scale and the opportunity to manipulate scaled objects should precede learning the related mathematics if large numbers are involved.

Another obstacle in teaching scale and ratio and proportion is finding an exciting context for the mathematics. Realizing that our middle school science colleagues were teaching the solar system while we were teaching scale, we hit upon the idea of enhancing their content with our mathematics. After all, who hasn't seen a bad model of the solar system, with disproportionate planet sizes and distances? Let the math teachers help!

Our goal was to develop a three-to-five-day unit that would refresh students' measurement skills, link those skills to the concepts of scale and proportion, and introduce the real-world applications of these concepts by using modeling and simulation to interest our students in our continued study of ratio and proportion. As part of the engineering design process, engineers use modeling and simulation after they define problems and generate and evaluate possible solutions. They assess their solutions by building models that can be tested repeatedly under a variety of simulated circumstances until the engineers can confirm the success or failure of the model. The benefit is that there is no cost or danger in running a simulation hundreds, thousands, even millions of times. This is not a new idea to students, who have played with toys and know that breaking a toy does not have the same consequences as breaking its real counterpart.

A very basic understanding of models is that they are replicas of real objects that are too big, too small, too costly, too remote, or too dangerous to manipulate and study effectively. Simulations are scenarios that allow for changes in variables and can be repeated numerous times so as to produce various outcomes. This repetition allows researchers to discover a range of possible outcomes, some desirable and some not so desirable.

We designed activities that build upon each other and link what students already know about measurement to how they can use measurement as a tool for creating scale models. The unit includes an introductory activity on measurement, two exploration activities, and a final demonstration of students' understanding of scale models. It introduces students to a teacher-made simulator developed in a spreadsheet that they can use to explore scale without getting overwhelmed with manipulating the vast distances in our solar system.

FIGURE 1 Standards-related lesson goals

	Mathematics	Science	Engineering
Big idea	Ratio and proportion	Earth's place in the universe	Engineering design process
Topic	Scale	The Earth and solar system	Modeling and simulation
Middle-grade standards	**Measurement:** Solve problems involving scale factors using ratio and proportion. **Connections:** Recognize and apply mathematics in contexts outside of mathematics.	The solar system consists of the Sun and a collection of objects, including planets that are held in orbit around the Sun. Patterns in the sky can be observed, described, predicted, and explained with models.	Engineering design involves a set of steps, which can be performed in different sequences and repeated if needed. Modeling, testing, evaluating, and modifying are used to transform ideas into practical solutions.
Source	(NCTM 2000)	(NRC 2011)	(ITEA 2007)

FIGURE 2 Student simulator

Solar system simulator (This is an image of the spreadsheet students use to generate scale values.)

A	B	C	D	E	F	G	H	I
	Diameters km	**10** cm	**15** cm	**20** cm	**25** cm	**Free choice** cm	**Challenge #1** cm	**Challenge #2** cm
Sun		0						
Mercury		0	0	0	0	0	0	0
Venus		0	0	0	0	0	0	0
Earth		0	0	0	0	0	0	0
Mars		0	0	0	0	0	0	0
Jupiter		0	0	0	0	0	0	0
Saturn		0	0	0	0	0	0	0
Uranus		0	0	0	0	0	0	0
Neptune		0	0	0	0	0	0	0
Pluto (dwarf planet)		0	0	0	0	0	0	0
Betelgeuse (red giant)		0	0	0	0	0	0	0
Sirius B (white dwarf)		0	0	0	0	0	0	0

Instructions

To build a model showing the relative sizes of the planets in our solar system and several other interesting heavenly bodies, you will start by developing an understanding of the relative sizes of each to the Sun.

1. Find and insert the diameter in kilometers in column B of each body listed.
2. Investigate the relative scale sizes of each body listed when you start with a Sun that is 10 cm in diameter (about the size of a softball). Insert a 10 in Column C for the Sun's diameter and watch what happens to the rest of column C. Use Record Sheet 1 (Figure 4) to record the diameters and determine how many of the resulting diameters you can draw on a sheet of notebook paper.
3. Do the same for columns D, E, and F, using 15, 20, and 25. Use Record Sheet 1 to record.
4. Column G is your chance to use the simulator over and over again. Choose any size for your Sun, put it in Column G, and watch the table calculate the relative sizes of the other bodies for your entire system. Do this a number of times until you have determined the set of measurements you would like to use in building a model solar system. Record these numbers and make a list of what common objects you might use to show the size calculated—balls, melons, oranges, grains of sand, etc. Use Record Sheet 2 (Figure 5) to record your attempts and your final choice of measurements.
5. Challenge 1: Use the simulator to help you determine what diameter your model of the Sun would have to be in order to use a 5 cm–diameter object to represent Pluto. What does this do to your model for Betelgeuse?
6. Challenge 2: Use the simulator to help you determine what diameter your model of the Sun would have to be in order to use a 10 cm–diameter object to represent Jupiter. What does this do to your model of Betelgeuse?

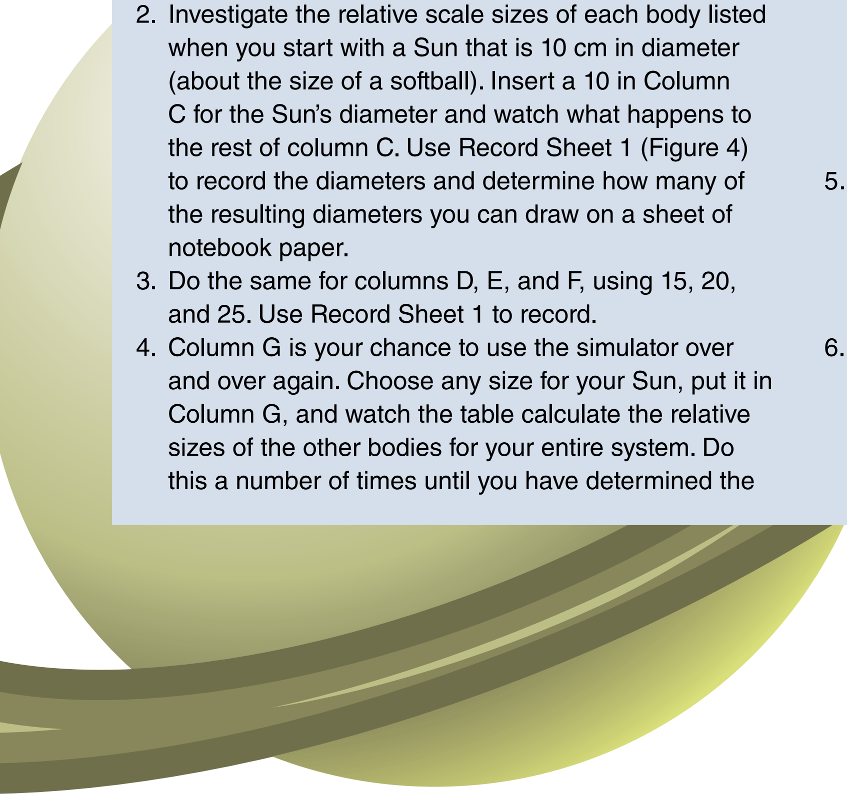

FIGURE 3 Teacher notes—Simulator

A	B	C	D	E	F	G	H	I
	Diameters km	**10** cm	**15** cm	**20** cm	**25** cm	**Free choice** cm	**Challenge #1** cm	**Challenge #2** cm
Sun	1,391,900						1,880	49
Mercury	4,866	=C5*0.0035						
Venus	12,106	=C5*0.0087						
Earth	12,742	=C5*0.0092						
Mars	6,760	=C5*0.0049						
Jupiter	142,984	=C5*0.1027						5.0323
Saturn	116,438	=C5*0.0837						
Uranus	46,940	=C5*0.0337						
Neptune	45,432	=C5*0.0326						
Pluto (dwarf planet)	2,274	=C5*0.0016					3.008	
Betelgeuse (red giant)	521,962,500	=C5*375						
Sirius B (white dwarf)	13,919	=C5*0.01						

Notes

1. All calculations derived on the teacher spreadsheet are based on the kilometers entered in column B. Be flexible in assessing students' tables, as their inputs might differ slightly from the ones used in this table. The constants in the formulas in column C are the ratio between each object and the Sun using the measurements shown.
2. The approximate answers to the two challenge questions are 1,880 (column H) and 49 (column I).
3. To build this simulator, insert the formulas shown in column C and extend across all columns.

Like all teachers, we are bound by the requirements of our state standards, which are based on national documents. In this lesson, however, we sought to move out of our comfort zone and include standards that addressed the science and engineering we were incorporating in our math lesson. Figure 1 provides a glimpse of the ideas that guided our lesson development.

Preparation

To prepare for this unit, we used spreadsheet software to develop a simple simulation based on the planets' diameters relative to the Sun. The purpose is to demonstrate to students that large objects can be scaled down to manageable and understandable sizes and that relative proportionality is maintained in scaling. We want them to experience this without the challenge of the mathematics of very large numbers getting in the way of the larger concept. Students can insert a diameter for the Sun in a measure they comprehend and the simulator will calculate the relative sizes of the planets in our solar system. In our simulator, students do not need to know the mathematics (we will teach that later) to understand the outcomes. To build a model representing the relative sizes of the bodies in the solar system, middle school students should be able to find appropriate objects based on the diameters calculated by the simulator. The student version (what students will see on the computer screen) of our solar system simulator is shown in Figure 2.

Initially, the simulator contains all zeros, because there are no inputs for the diameter of the Sun. Once students input a value for the diameter of the Sun, e.g., 10 cm, the rest of the column populates with the relative scale values for the rest of the bodies. The teacher's version of the simulator, with notes and formulas, is shown in Figure 3. Building this tool requires minimal skill with

FIGURE 4 Record sheet 1

Solar system scale recording sheet

A	B	C	D	E	F	G	H
	Diameters km	**10** cm	**15** cm	**20** cm	**25** cm	**Challenge #1** cm	**Challenge #2** cm
Sun							
Mercury							
Venus							
Earth							
Mars							
Jupiter							
Saturn							
Uranus							
Neptune							
Pluto (dwarf planet)							
Betelgeuse (red giant)							
Sirius B (white dwarf)							

Refer to the simulator directions to complete the table above.

1. Did you have problems trying to draw lines on notebook paper to represent diameters when using the numbers generated in columns C, D, E, and F? Explain.
2. **Challenge #1:** What diameter of the Sun gives you a 3 cm diameter for Pluto? ________________cm
   How many entries did you try? ________________
3. **Challenge #2:** What diameter of the Sun gives you a 5 cm diameter for Jupiter? ________________cm
   How many entries did you try? ________________

spreadsheet software, specifically, inserting formulas and a text box with instructions. Once the tool is built, it can be copied into word-processing software to create a variety of recording sheets for students. Using the formulas provided, the simulator can be re-created in about 15 minutes in any spreadsheet. Copying the spreadsheet to any word-processing document takes only seconds.

Classroom activities

Day 1

To prepare our students for the idea that measurement would be our context for exploring ratio and proportion, we planned a scavenger hunt to reintroduce measuring skills and vocabulary, a video review of ratio and proportion, and a quick challenge. You can design the scavenger hunt to fit your own purposes and circumstances. Our design has three phases. First, distribute meter sticks to teams of students and have them measure some common classroom objects (chalk, desktop, textbook, floor tiles, room length) and listen for the student dialogue to include the correct use of metric terms. If your meter sticks have English measures on the flip side, require students to use the metric side and to discuss their findings in metric terms. Have students quickly call out their findings and verify their results. Next, collect the meter sticks and distribute a three-column record sheet with preassigned metric measurements in the first column. In the second column, direct students to list objects in the classroom (or in the hall or playground) that they think match the measurements. When they are finished, redistribute the meter sticks and have teammates verify their answers with notations in the third column. Since students are mobile during this assignment, we recommend allowing about 30 minutes to complete it.

After initiating the conversation about measurement, introduce the concept of scale to students using one of NASA's eClips, "Real World: Scale Models and Ratios" (see Resources). This short video clip uses a number of examples to present the purpose and usefulness of ratio and proportion in determining scale. The mathematics introduced in the video involves the use of cross products to find a missing variable, and we reinforce this skill with simple toys. We end the lesson by passing out various dolls to student teams and challenging them to determine the scale used to make the doll by comparing the height of the doll to the height of one of their teammates. Answers will vary, but all teams should be able to take the lesson from the video and apply it to the task. At a minimum, we expect them to determine that 1" on the doll represents X" on their chosen teammate. If time permits, they can explore ratios between arm and leg length, hand width, and head circumference.

The measurement scavenger hunt is intended to be formative, letting us know if or how much we need to review metric measurements and equivalent fractions. Finding the scale factor between a doll and a human is

FIGURE 5 Record sheet 2

Solar system model recording sheet

A	B	C	D	E	F	G	H
	Diameters km	**Free choice** cm	**Free choice** cm	**Final choice** cm	**Object to use**	**Distance from Sun** (orbit radius)	**Scale from Sun** (orbit radius)
Sun							
Mercury							
Venus							
Earth							
Mars							
Jupiter							
Saturn							
Uranus							
Neptune							
Pluto (dwarf planet)							

Instructions

1. With the class simulator, play with the free-choice option using as many measurements as you like. Record any two tries on the record sheet and then record one more of your outcomes to use for building your model of the solar system.
2. Suggest examples of objects that represent the sizes you select for your model.
3. Save these numbers as you use the online simulator to calculate the distances between the planets.
4. Go to *www.exploratorium.edu/ronh/solar_system*.
5. In column G, record the actual orbits in km that are given in this simulator. Enter your selected diameter for the Sun. Note that you have been working in cm and that the Exploratorium simulator uses mm. The simulator will generate all of the related diameters and radii of each orbit. Record these radii in the table. (Check the diameters given with yours.)
6. Think about the orbital scale radii generated by the simulator and describe how you will represent these measurements in your model.

a check for understanding. It lets us know if students' concept of scale is sufficient to move on to the next day's activities. If not, we recommend starting the next day with a review of equivalent fractions and relating this to the doll–human scale challenge.

Days 2 and 3

On days 2 and 3, we use two visually impressive YouTube clips to illustrate the relative size of planets (see Resources) and ask students to suggest reasons why it is so difficult to build a truly accurate model of our solar system. (If YouTube is blocked at your school, you can download the videos to a thumb drive at home and show them on a class computer.) After a brief discussion of the relative size of some of the bodies in the videos, we show students how to use the simulator we developed for this lesson and have them record their results on Record Sheet 1 (Figure 4).

The questions that accompany the simulator guide students' exploration options. Record Sheet 1 is a hard copy of the simulator's columns and rows, which allows students to document the answers generated by the simulator. It can be used to assess how students responded to questions 1 through 5 listed below the simulator. The advantage of the simulator is that students can repeatedly enter numbers to determine reasonable responses to the challenge questions. As mentioned earlier, they will begin to realize that a simulation allows for changes in variables and can be repeated numerous times so as to produce various outcomes.

Days 3 and 4

On days 3 and 4, we let students investigate an online simulator (see Resources) that can scale the distances in and beyond our solar system, and we challenge students to develop a model that incorporates both accurate scale sizes of the planets and scale distances between them. As they record their results on Record Sheet 2 (Figure 5), they will be amazed at the distances between the planets.

Record Sheet 2 allows students to manipulate the free-choice column in the simulator. While they can tinker endlessly with a variety of inputs, they must eventually decide on a series of manageable measurements on which to base their solar system model. Because our simple classroom simulator deals only with the diameters of the planets, we use the online simulator to calculate scale distances from the Sun that are matched to the diameters of the planets. Students have been working with centimeters, but the online simulator requires inputs in millimeters. This affords us an opportunity to revisit metric conversions and relate the process to ratio and proportion, if necessary.

Day 5

On the last day of the unit, students demonstrate their models outside using the objects and distances determined with the simulators and documented on Record Sheet 2. Be prepared to go out on the football field or the parking lot and bring binoculars! This visual representation of a true model of the solar system will drive home the concepts of relative size and vast distances, and the use of scale modeling to demonstrate both. This should lay the groundwork for delving into the mathematics of ratio and proportion, as students will want to know how the simulators actually work. As a side goal, they could eventually start building their own simple simulators.

Assessment and student outcomes

There are multiple opportunities for both formative and summative assessments throughout this unit. As mentioned earlier, the Day 1 activities give the teacher a good feel for students' prior knowledge and skill level for exploring the concepts of scale and ratio and

proportion. The record sheets help us determine how well students are manipulating the simulator. These sheets also are formative in that we can always pose more questions and challenges if we sense that students are struggling.

We have two summative assessments planned to evaluate students' understanding of the concept of scale. First, we will rate the models they build and demonstrate by verifying that the diameters and distances are accurate representations of the solar system. Then, after some direct skill-building work with ratio and proportion equations, we will use solar system images from Image Scale Math (Odenwald and Lewis), a NASA resource available online that was designed as a supplement for teaching mathematical topics. By comparing the size of the image to the actual size of the feature (Moon craters, mountain ranges), students are expected to determine the scale factor used to reduce the image to the page. This resource allows us to maintain the space theme in our lesson and check for students' ability to manipulate the mathematics of scale.

Lessons learned

We found that the more we sought out ideas for infusing modeling and simulation into our lessons, the more resources we found. We realized that we did not have to develop our own simulators (although we are proud that we challenged ourselves to do so), we just needed to learn how to search for online examples that are relevant to our lessons. For example, we decided that we can extend our study of scale into geometry and incorporate several resources on scaling buildings (see Resources).

We also learned that we can give our students a little more control over their own learning with models and simulations. Students can tinker repeatedly with number combinations and outcomes without getting lost in the calculations and can understand big ideas before learning the underlying mathematics. We believe that learning the big ideas will help our students appreciate and want to explore the related mathematics.

Our plan is to test our lesson during the current school year and refine it as we observe how our students respond to it. If you would like to adapt our simulator and worksheets for your own classroom, email us and we will send you the spreadsheet simulator and related lesson materials. Again, let the math teachers help! ■

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Resources

Build a solar system—*www.exploratorium.edu/ronh/solar_system*

Real world: Scale models and ratios—*www.youtube.com/watch?v=IYizIhPvMWQ*

Scaling the pyramids—*www.pbs.org/wgbh/nova/pyramid/geometry*

Star size comparison HD—*www.youtube.com/watch?v=HEheh1BH34Q*

STR's skyscraper models—*www.skyscrapercity.com/showthread.php?t=158364*

Universal perspective & the relative size of planets.—*www.youtube.com/watch?v=wyC99HbmuY8*

Karen Charles (*kcharles@rti.org*) facilitated the NASA Academy in Kingsville, Texas. She is a research education analyst at RTI International in Durham, North Carolina. **J.D. Canales, Angela Smith**, and **Natalie Zimmerman** are mathematics teachers at Memorial Middle School in the Kingsville Independent School District in Kingsville, Texas