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LIGHT TRANSMITTANCE ACROSS SMOKE PLUMES DOWNWIND FROM POINT SOURCES OF AEROSOL EMISSIONS

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Abstract—A new calculation procedure for theoretically predicting the magnitude of light transmittance across a smoke plume downwind of its emission source is presented. The calculation procedure is based on a combination of Mie light scattering theory and the Gaussian (Pasquill–Gifford) plume diffusion equations. Source information needed to estimate the light transmittance through the plume at a given downwind include the meteorological conditions, stack height, gas exhaust temperature and velocity, aerosol emission mass flowrate, size distribution, and refractive index. Graphs of the horizontal plume light transmittance dispersion parameter and the horizontal average extinction coefficient dispersion parameter are presented as a function of the downwind distance. A sample calculation is included to illustrate the procedure and use of the various parameters.

NOMENCLATURE

- b light extinction coefficient (m^{-1})
- $\bar{b}$ mean light extinction coefficient across the plume (m^{-1})
- C correction factor for averaging time
- H height of emission source (m)
- I light flux
- I_0 incident light flux
- K ratio of specific particulate volume to extinction coefficient ($cm^3\ m^{-3}\ m^{-1}$)
- l path length
- L total path length
- Q source emission ($g\ s^{-1}$)
- t averaging time
- T transmittance I/I_0
- u wind speed ($m\ s^{-1}$)
- W pollutant mass concentration ($g\ cm^{-3}$)
- x downwind distance (km)
- x' distance of the observer from the source (km)
- y horizontal perpendicular distance relative to the center of the plume
- z vertical distance.

Greek

- η_y plume extinction coefficient dispersion parameter along y plane (m^{-2})
- η_z plume extinction coefficient dispersion parameter along z plane (m^{-2})
- ξ_y plume transmittance dispersion parameter along y plane (m^{-1})
- ξ_z plume transmittance dispersion parameter along z plane (m^{-1})
- ρ average particle density ($g\ m^{-3}$)
- σ_y plume dispersion coefficient y direction
- σ_z plume dispersion coefficient z direction.

1. INTRODUCTION

1. *Plume visual effects*

Plumes can cause safety hazards if they drift over highways or airports by limiting the vision of motorists or pilots. The impact of smoke plumes from nearby sources on the operation of an airport is described in the *Kansas City, Kansas—Kansas City Missouri Air Pollution Abatement Activity Report* (1967).

The visibility and the visual obscuration of objects by the plume are two visual effects caused by smoke plumes. The terms "visibility" and "visual range" are often used interchangeably to mean the longest distance between the observer and an object that is visually detectable. In this paper "visibility" as applied to plumes simply means the clearness with which objects stand out from their surroundings under good visual conditions, as suggested by the originator of both terms and reported by Middleton (1952). Conner and Hodkinson (1967) have studied the appearance of smoke plumes near the source under controlled conditions and reported that the important variables include background illumination; the relative position of the sun, plume, and the observer; the physical properties of the aerosol; and the plume size. Variations in plume appearance, particularly for white plumes, are caused by changes in the amount of light scattered by the plume and the amount of light transmitted through the plume towards the observer. Conner and Hodkinson reported that the plume transmittance T (the fraction of light passing through the plume) is an optical property intrinsic to a specific plume. In the absence of quantitative data concerning the background illumination (angle of illumination, intensity of background light, intensity of light scattered by clouds, and the contrast of the viewing background to the plume) the light transmittance can be used to provide an estimate of both the plume visibility (whether or not the plume can be seen) and plume opacity (magnitude by which one can see through the plume). When the light transmittance through the plume is close to 100 per cent, the plume is probably invisible. When the light transmittance is near zero, the plume is easily seen and visibility through the plume is impossible.

Jarman and de Turville (1966) reported an empirical relationship between plume transmittance, plume to background contrast, and the screening of objects by the plume.

The prediction of the size of the envelope of smoke (the outer boundaries of the smoke plume) has been reported by Roberts (1923), Sherwood (1947) and many others. The physical dimensions of the visible plume (length and width) has also been used to calculate the atmospheric dispersion coefficients as described by Gifford (1959). Jarman and de Turville (1969) also reported theoretical predictions for plume disappearance for an observer standing under the plume looking skywards based on light scattering and plume dispersion theories. The light transmittance of a plume some distance from its sources was estimated for elevated point sources near an airport in the *Kansas City, Kansas—Kansas City Missouri Air Pollution Abatement Activity Report* (1967) and for open burning near highways by Duckworth (1965).

2. *Objective*

The objective of this paper is to present a systematic method for estimating the light transmittance horizontally through smoke plumes downwind of their sources.

The information provided by this approach can be used to estimate the visual effects of new and existing sources of air pollution.

2. THEORY

1. *Light attenuation*

The change in the light flux I, from single scattering of light as it passes through an aerosol cloud of thickness dl is given by

$$\frac{dI}{I} = -b(l)\,dl \tag{1}$$

where $b(l)$ is the light extinction coefficient (m^{-1}) which may be a function of position l. If the spatial variations of the extinction coefficient $b(l)$ are due solely to spatial variations in the aerosol mass concentration $W(l)$ ($g\,m^{-3}$), the extinction coefficient can be written in a form reported by PILAT and ENSOR (1970) as

$$b(l) = \frac{W(l)}{\rho K} \tag{2}$$

where the aerosol concentration $W(l)$ is averaged over all particle sizes, ρ is the density of the particles ($g\,cm^{-3}$), and K is the ratio of the specific particle volume to extinction coefficient ($cm^3\,m^{-3}\,m^{-2}\,m^{-3}$). K is a function of the particle size distribution and the particle refractive index. K can be calculated from the light transmittance, illumination path length, and mass concentration at the emission source as outlined by PILAT and ENSOR (1970). Some representative average values of K are presented in TABLE 1. If the particle size distribution and the refractive index of the particles do not change in the plume, K can be used to estimate the optical properties of the plume both at the source and some distance from the source. The similarity between the magnitudes of Ks of emission sources and of the atmospheric aerosol, as reported by PILAT and ENSOR (1971), implies that a plume K may be assumed to be reasonably constant.

TABLE 1. EMISSION SOURCE OPTICAL FACTORS

Source	Average K ($cm^3\,m^{-2}$)	Reference
White smoke (vaporized oil)	0·30	CONNER and HODKINSON (1967)
Black smoke (fuel oil)	0·06	CONNER and HODKINSON (1967)
Black smoke (coal stoker)	0·10	STOECKER (1950)
White smoke (kraft recovery furnace)	0·10	LARSSEN, ENSOR and PILAT (1972)

The light transmittance T of a plume of particles with spatial variations in mass concentration can be calculated by substituting equation (2) for the extinction coefficient $b(1)$ into equation (1) and integrating across the optical pathlength L

$$\ln T = \ln (I/I_0) = \frac{-1}{\rho K} \int_0^L W(1)\, d1 \tag{3}$$

where I_0 is the light flux incident on the plume.

A mean extinction coefficient $\bar{b}$ may be defined for the plume such that

$$\ln (I/I_0) = -\bar{b}\, L \tag{4}$$

Equations (3 and 4) can be combined to give

$$\bar{b} = \frac{1}{L\rho K} \int_0^L W(1)\, d1. \tag{5}$$

Equations (3 and 5) can be used to estimate the optical properties of smoke plumes downwind from point sources, if both the optical path length L (also the plume diameter) and $\int_0^L W(1)\, d1$ are known or if they can be calculated. Both the optical path length (plume dia.) and $\int_0^L W(1)\, d1$ can be estimated by the use of plume dispersion equations. Also, the spatially averaged light extinction coefficient $\bar{b}$ may be useful for estimating the meteorological range through the plume as defined by MIDDLETON (1952).

2. *Dispersion of smoke plumes*

The dispersion of plumes from point emission sources has been studied extensively during the past 30 y. A summary of this research and the information concerning the magnitude of the vertical and horizontal plume diffusion coefficients σ_y and σ_z, respectively, was reported by TURNER (1969). The mass concentration of a pollutant in the plume is given by the Gaussian (Pasquill–Gifford) plume dispersion equation:

$$W(x, y, z, H) = \frac{Q}{2\pi\sigma_y\sigma_z u} \exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right]\left\{\exp\left[-\frac{1}{2}\left(\frac{z-H}{\sigma_z}\right)^2\right] + \exp\left[-\frac{1}{2}\left(\frac{z+H}{\sigma_z}\right)^2\right]\right\} \tag{6}$$

where Q is the pollutant emission rate at the source, u the windspeed, y and z the horizontal and vertical distance, respectively as shown in FIG. 1, x is the downwind distance from the emission source; and H is the effective emission height (or vertical height of plume centerline).

3. *Effect of smoke plumes on the atmospheric visual quality downwind of emission sources*

The effect of a smoke plume on the visual quality of the atmosphere downwind of the smoke source can be substantial. Let us consider the case of an observer looking through the plume in a direction perpendicular to the downwind axis of the plume

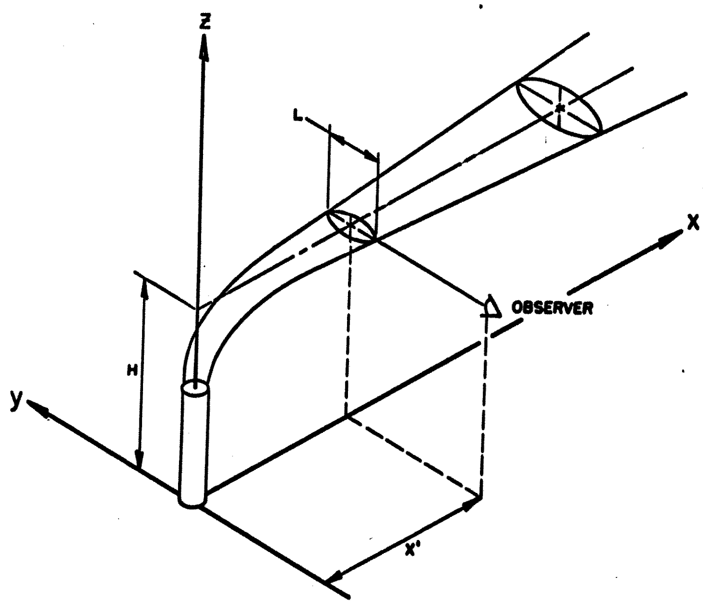

FIG. 1. Dispersion of smoke from a point source.

as shown in FIG. 1. This situation corresponds to an observer viewing the plume from a location at the same elevation as the plume (from possibly a hill or tower). The viewing path in a direction perpendicular to the plume downwind axis (the x axis) for the maximum effect on the visual quality (light transmittance through the plume) is along the path of maximum smoke concentration and maximum plume path length. The visual path that meets this criteria is through the plume centerline. If the assumptions concerning plume dispersion are valid, the vertical height of the plume centerline is the effective stack height H ($z = H$). If this value of z is substituted for z in equation (6)

$$W(x, y, z = H, H) = \frac{Q}{2\pi\sigma_y\sigma_z u}\exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right]\left[1 + \exp\left(-\frac{2H^2}{\sigma_z^2}\right)\right]. \quad (7)$$

Equation (7) can be integrated across the horizontal plume dia. $4\sigma_y$ and substituted into equation (3) to calculate the light transmittance across the plume

$$\ln\left(\frac{I}{I_0}\right)_y = -\frac{Q[1 + \exp(-2H^2/\sigma_z^2)]}{u\rho K 2\pi\sigma_y\sigma_z}\int_0^{4\sigma_y}\exp[-\tfrac{1}{2}(y/\sigma_y)^2]\,dy \quad (8)$$

where σ_y and σ_z are evaluated at some downwind distance $x = x'$. The integral is similar to the error integral,

$$\int\left(\frac{1}{\sqrt{2\pi}}\right)e^{-0.5p^2}\,dp,$$ which has the property that

$$\int_{-2}^{+2}\left(\frac{1}{\sqrt{2\pi}}\right)e^{-0.5p^2}\,dp \approx 1. \quad (9)$$

If the variables in equation (8) are transformed as indicated below

$$p = y/\sigma_y,\ y = \sigma_y p$$

$$\mathrm{d}p = \mathrm{d}y/\sigma_y,\ \mathrm{d}y = \sigma_y \mathrm{d}p$$

and equation (9) is integrated over the limits $-2\sigma_y$ to $+2\sigma_y$ (physically the same as $4\sigma_y$) equation (8) becomes

$$\ln\left(\frac{I}{I_0}\right)_y = \frac{-Q[1 + \exp(-2H^2/\sigma_z^2)]}{u\rho K 2\pi\sigma_y\sigma_z}\int_{-2\sigma_y}^{+2\sigma_y} e^{-0.5p^2}\sigma_y\,\mathrm{d}p \tag{10}$$

or from the properties of the error integral

$$\ln\left(\frac{I}{I_0}\right)_y \approx \frac{-Q}{u\rho K\sqrt{(2\pi)}\,\sigma_z}[1 + \exp(-2H^2/\sigma_z^2)]. \tag{11}$$

The analysis may be simplified by grouping all the plume dispersion variables together and setting them equal to a plume transmittance dispersion parameter

$$\xi_y = \frac{1}{\sqrt{(2\pi)}\,\sigma_z}[1 + \exp(-2H^2/\sigma_z^2)] \tag{12}$$

where ξ_y is a function of the meteorological stability and the downwind distance. The downwind light transmittance T_y at a distance x' from the source is

$$T_y = (I/I_0)_y = \exp\left(-\frac{\xi_y Q}{\rho K u}\right) \tag{13}$$

where Q, ρ, and K are all functions of the source.

The mean light extinction coefficient $\bar{b}$ defined in equation (4) can be calculated from equation (11).

$$\bar{b}_y = \frac{Q}{4\,u\rho K\sqrt{(2\pi)}\,\sigma_y\sigma_z}[1 + \exp(-2H^2/\sigma_z^2)]. \tag{14}$$

The variables that are a function of the plume dispersion are grouped into a light extinction coefficient dispersion parameter

$$\eta_y = \frac{1}{4\sqrt{(2\pi)}\,\sigma_y\sigma_z}[1 + \exp(-2H^2/\sigma_z^2)]. \tag{15}$$

The mean extinction coefficient is given by

$$\bar{b}_y = \frac{\eta_y Q}{\rho K u}. \tag{16}$$

JARMAN and DE TURVILLE (1969) reported a similar analysis for the case of an observer looking vertically through the plume along the z axis. The resulting equations, modified to include light transmittance and the mean extinction coefficient are

$$\xi_z = \frac{1}{\sqrt{(2\pi)}\,\sigma_y} \tag{17}$$

$$\eta_z = \frac{1}{4\sqrt{(2\pi)}\,\sigma_y\sigma_z}. \tag{18}$$

3. RESULTS AND DISCUSSION

The horizontal plume extinction dispersion parameter η_y and the horizontal plume transmittance dispersion parameter ξ_y are presented as a function of the downwind distance with various effective stack heights for meteorological stability conditions D (neutral), E, and F in FIG. 2–7. Examination of these figures shows that both η_y and ξ_y decrease with downwind distance, thus the light transmittance is always increasing with dilution of the plume. This indicates that the reduction in the plume aerosol mass concentration has a greater effect than the increase in the illumination pathlength. The plumes emitted aloft (from smoke stacks where $Z > 0$) have identical curves of ξ_y and η_y versus the downwind distance until the plume reaches the ground. Then the aerosol is reflected back into the plume and the curves approach the ground level case. The curves for the horizontal plume extinction dispersion parameter η_y are quite similar in form to those for the horizontal plume transmittance dispersion parameter ξ_y. It should be noted that $\eta_y\,(H = 0) = \eta_z$.

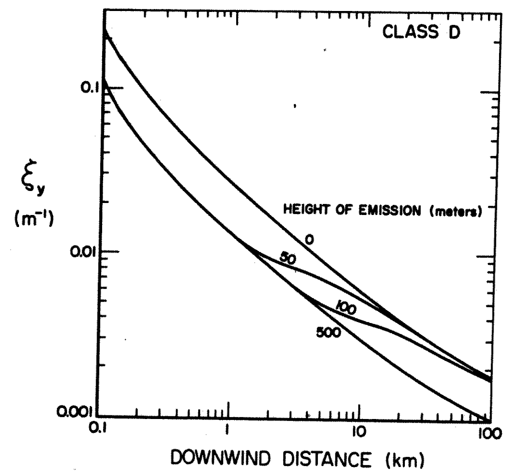

FIG. 2. Plume transmittance dispersion parameter as a function of downwind distance for stability class D.

These calculations combine plume dispersion and optical effects which usually have different time averages. The estimated optical effects are for 10 min averages corresponding physically to a time exposure photograph (GIFFORD, 1959). The stability classes have been limited to stable conditions where the visibility impact is most severe and the plume exhibits relatively restrained motion. If the averaging time is changed, both ξ_y and η_y must be multiplied by a correction factor C to correct for the new averaging time, t. Based on the results reported by TURNER (1969), the correction factor is given by

$$C \propto t^{-0.17} \tag{19}$$

For an averaging time of 1 s, reported by JARMAN and DE TURVILLE (1969) as reasonable for visual observations, the correction factor is about 3.

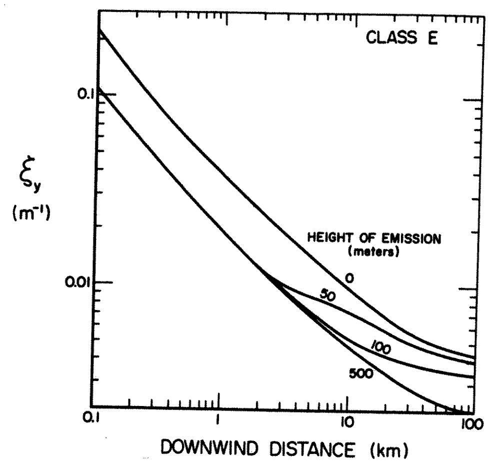

FIG. 3. Plume transmittance dispersion parameter as a function of downwind distance for stability class *E*.

The use of the graphs to calculate the light transmittance across a plume downwind of its emission source is demonstrated by the following example. The problem is to predict the light transmittance through the plume at downwind distances of 0·1 and 1·0 km at meteorological conditions of 5 m s^{-1} wind speed and class *D* atmospheric stability. The aerosol source emisssion parameters are a source emission

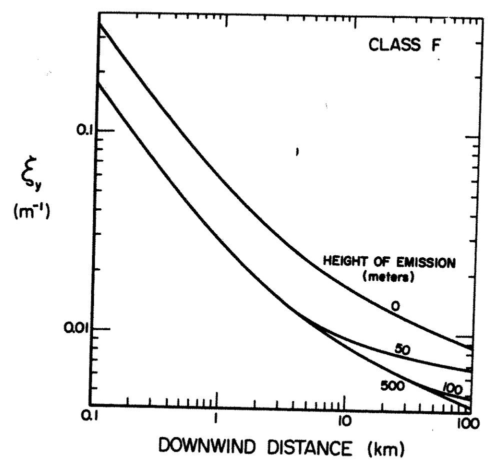

FIG. 4. Plume transmittance dispersion parameter as a function of downwind distance for stability class *F*.

height of 100 m, particle emission rate of 1 g s^{-1}, particle density of 2·0 g cm^{-3}, and *K* of 0·06 cm^3 m^{-2} (Black smoke in TABLE 1). From FIG. 2 the plume transmittance dispersion parameters at 0·1 and 1·0 km are obtained

$$\xi_y(0{\cdot}1 \text{ km}) = 1{\cdot}02 \text{ m}^{-1}$$

$$\xi_y(1{\cdot}0 \text{ km}) = 0{\cdot}14 \text{ m}^{-1}.$$

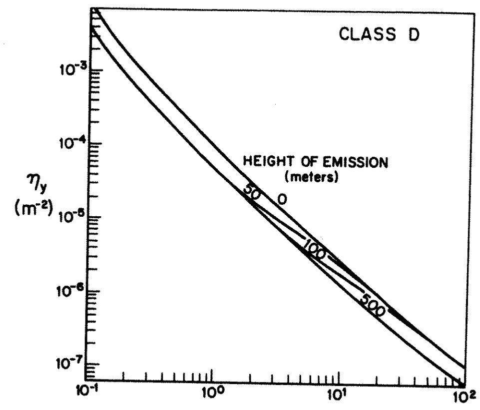

FIG. 5. Plume extinction coefficient dispersion parameter as a function of downwind distance for stability class *D*.

The light transmittance across the plume is calculated by substituting the proper parameters into equation (13). At a downwind distance at 0·1 km

$$T_y = \exp - \left[\frac{(1.02\ m^{-1})(1\ g\ s^{-1})}{(2\ g\ cm^{-3})(0.06\ cm^3\ m^{-2})(5\ m\ s^{-1})} \right] = 0.18$$

and at 1.0 km

$$T_y = \exp - \left[\frac{(0.14)(1)}{(2)(0.06)(5)} \right] = 0.79.$$

Thus for this example problem the light transmittance through the plume centerline at the downwind distances of 0·1 and 1·0 km are 18 and 79 per cent, respectively. The effective emission height may be estimated by using appropriate plume rise equations as summarized by TURNER (1969) or BRIGGS (1969). The source optical parameter K may be estimated from the aerosol size distribution and refractive index

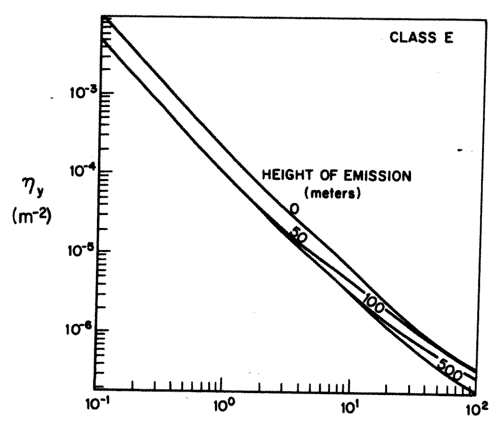

FIG. 6. Plume extinction coefficient dispersion parameter as a function of downwind distance for stability class *E*.

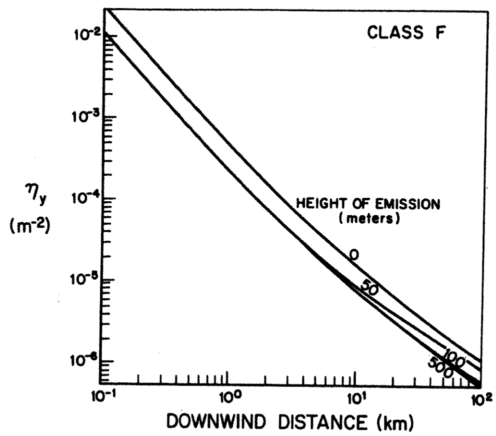

FIG. 7. Plume extinction coefficient dispersion parameter as a function of downwind distance for stability class *F*.

or from the aerosol mass concentration and plume opacity measurements at the source, as reported by PILAT and ENSOR (1970).

These theoretical predictions of the light transmittance horizontally through the plume centerline downwind of the source have not been compared to field measurements. However, it should be noted that these predictions are based on a combination of the Mie light scattering theory and the Gaussian (Pasquill–Gifford) plume dispersion equation, both of which have had considerable experimental verification.

4. CONCLUSION

A new procedure for predicting the light transmittance horizontally through the centerline of a smoke plume downwind of its emission source is presented. Graphs of the plume transmittance dispersion parameter versus the downwind distance from the plume at various emission heights and atmospheric stability classes enable the light transmittance across the plume to be calculated from the aerosol source properties (emission height, particle density, particle mass emission rate, particle size distribution). This new procedure should be useful in estimating the effect of source emissions on the plume opacity (and atmospheric visibility) downwind of the source.

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REFERENCES

BRIGGS G. A. (1969) *Plume Rise.* AEC Critical Review Series, U.S. Atomic Energy Commission, TID-25075.

CONNER W. D. and HODKINSON J. R. (1967) *Optical Properties and Visual Effects of Smoke Stack Plumes.* P.H.S. Publ. No. 999-AP-30.

DUCKWORTH S. (1965) The meteorologically scheduled open burn. *J. Air Pollut. Control Ass.* **15,** 274–277.

GIFFORD F. (1959) Smoke plumes as quantitative air pollution indices. *Int. J. Air Poll.* **2,** 42–50.

JARMAN R. T. and DE TURVILLE C. M. (1966) The screening power of visible smoke. *Int. J. Air Wat. Pollut.* **10,** 465–467.

JARMAN R. T. and DE TURVILLE C. M. (1969) The visibility and length of chimney plumes. *Atmospheric Environment* **3,** 257–280.

KANSAS CITY, KANSAS—KANSAS CITY, MISSOURI AIR POLLUTION ABATEMENT ACTIVITY REPORT (1967) U.S. Dept. HEW, PHS, NAPCA.

LARSSEN S., ENSOR D. S. and PILAT M. J. (1972) Relationship of plume opacity to the properties of particulates emitted from kraft recovery furnaces. *TAPPI* **55**, 88–92.

MIDDLETON W. E. K. (1952) *Vision Through the Atmosphere.* University of Toronto Press, Toronto.

PILAT M. J. and ENSOR D. S. (1970) Plume opacity and particulate mass concentration. *Atmospheric Environment* **4**, 163–173.

PILAT M. J. and ENSOR D. S. (1971) Comparison between the light extinction aerosol mass concentration relationship of atmospheric and air pollutant emission aerosols. *Atmospheric Environment* **5**, 209–215.

ROBERTS, O. F. T. (1923) The theoretical scattering of smoke in a turbulent atmosphere. *Proc. R. Soc.* **A104**, 640–654.

SHERWOOD T. K. (1949) The geometry of smoke screens. *J. Met.* **6**, 416–419.

STOECKER W. F. (1950) Smoke density measurement. *Mech. Engr* **72**, 793–798.

TURNER, B. D. (1969) *Workbook of Atmospheric Dispersion Estimates.* PHS Publ. No. 999-AP-26.